

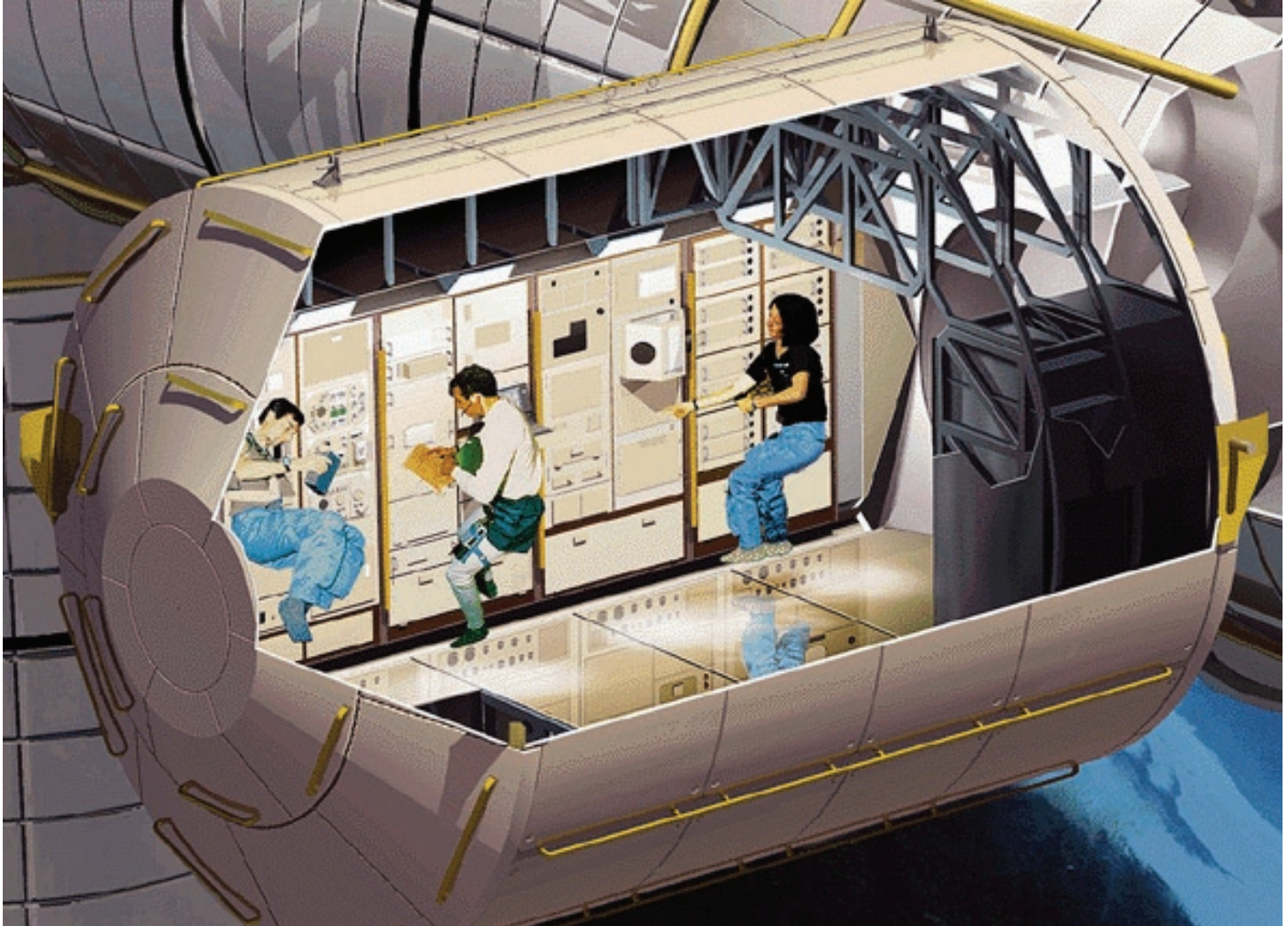
Mathematics

Surface Area, Volume, and Linear Dimensions of a Cylinder

Lesson 3

Grades 5-8

Page 29



International Space Station
Columbia Orbital Facility



Background

Scientists at NASA are building a robot called the Personal Satellite Assistant, or PSA. The PSA's mission is to keep the astronauts safe and to assist them with their chores on space-based vehicles such as the International Space Station (ISS), a Crew Exploration Vehicle, or even on Mars. This small round robot will float in microgravity and move autonomously (without direction from people). It will keep track of the astronauts' schedules, monitor supplies, assist with scientific experiments, communicate with Mission Control, and help keep the astronauts safe by monitoring the air composition and temperature.

NASA engineers have created a model of the PSA with a diameter of 30.5 centimeters (12 inches). The engineers' goal is to build a PSA with a 20-centimeter (8-inch) diameter, because it will be safer and will require less power to move around.





Introduction

In this lesson, students use calculations, visualization, and logical reasoning to discover that a wide cylinder has more surface area than a tall cylinder when their volumes are the same. Students will also find that a change in the diameter of a cylinder has a greater impact on surface area and volume than a change in height.



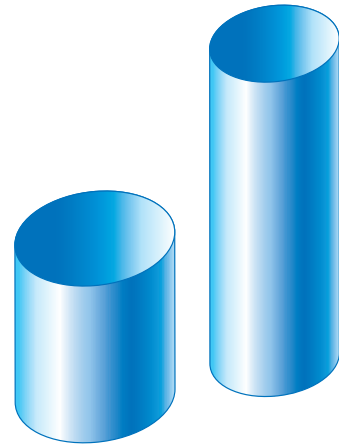
Main Concept

The surface area of a cylinder is greatest when its height-to-radius ratio is either very large or very small.



NASA Relevance

NASA scientists and engineers working on the PSA project need to reduce the volume and mass of the PSA because of the high cost of transportation for leaving our home planet, limited space in space-based vehicles, and for safety reasons. Even though they use computer programs to design the PSA and its components, a basic understanding of volume and surface area is essential in order to design the PSA and the shape of the components that go inside it.



Prerequisite Skills

Students should be able to:

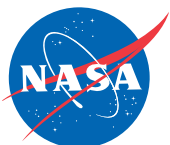
- Calculate the surface area and volume of a cylinder.
- Conduct basic math operations using decimals.
- Calculate and interpret ratios.
- Measure the height and radius of a cylinder.



Instructional Objectives

During this lesson, students will:

1. Explain that the surface area of a cylinder will change as the radius and height change, even when the volume remains the same.
2. Explain that the surface area of a cylinder is greatest when its height-to-radius ratio is either very large or very small.
3. Make a recommendation on which dimension of a cylinder in the PSA should be increased in order to maximize its surface area.



NATIONAL EDUCATION STANDARDS

Fully Met	Partially Met	Addressed
NCTM (6-8) <i>Geometry #1</i> <i>Measurement #2</i> 2061: 9B (6-8) #2	NCTM (6-8) <i>Geometry #4</i>	NCTM (6-8) <i>Measurement #1</i> 2061: 2B (6-8) #1 2061: 9C (6-8) #1 NSES E (5-8) #1

**Major Concepts**

- Some shapes have special properties.
- Mathematical statements can be used to describe how one quantity changes when another changes.
- There are many possible solutions to a problem.
- Ratios can be used to see mathematical relationships or to compare changes.

**Materials and Equipment**

- Computers with Internet connections (see table next page)
- Cylinders that are tall/narrow and short/wide (for demonstration)
- One clay cylinder (radius = 1 cm, height = 8 cm) per group
- 1 copy of the Student Handout sheet for each student or group of students
- Rulers or tape measures for each group
- Calculators (optional)





System Requirements to Run PSA Web Site Activities

Platform	Browser
Windows 95 Windows 98 Windows Me	Internet Explorer 4.0 or later (Internet Explorer 5.0 or later is recommended), Netscape Navigator 4 or later, Netscape 7.0 or later (Netscape 6 is not recommended) JavaScript enabled
Windows NT Windows 2000 Windows XP or later	Internet Explorer 4.0 or later, Netscape Navigator 4 or later, Netscape 7.0 or later, with standard install defaults (Netscape 6 is not recommended) JavaScript enabled
Macintosh: 8.6 thru 9.2	Netscape 4.5 or later (Netscape Communicator 4.7 or Netscape 7.0 are recommended), Netscape 7.0 or later, (Netscape 6 is not recommended) Microsoft Internet Explorer 5.0 or later JavaScript enabled
Macintosh OS X 10.1 or later	Netscape 7.0 or later (Netscape 6 is not recommended), Microsoft Internet Explorer 5.1 or later JavaScript enabled
Browser plug-ins	Flash Player 6 or higher QuickTime Player 6 or higher



Time for Activity

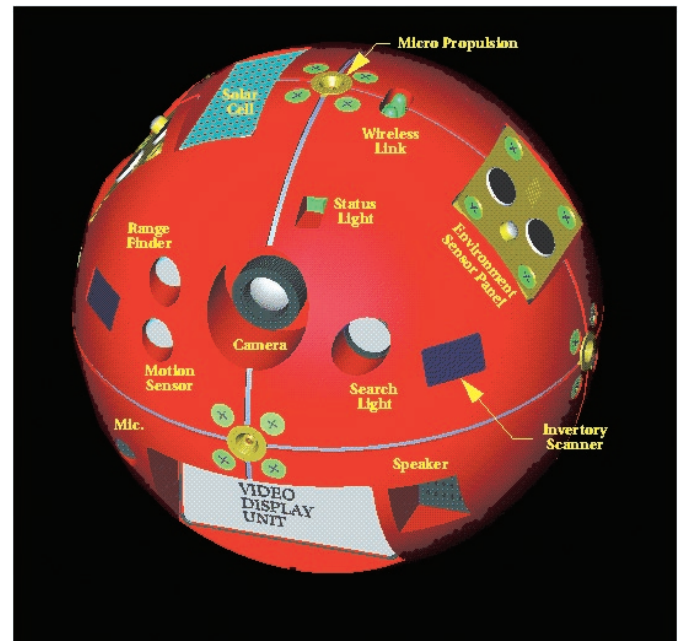
1 class period



LESSON Engage

Remind students that NASA's PSA robot will require many different components that are not only shaped like rectangular prisms but also cylinders. Go to the PSA Web site at <http://psa.arc.nasa.gov/>. In the "Activities" section, have the students visit the "PSA Systems" and look at the PSA images. Ask students what objects on the PSA are cylinders. Students may recognize the cameras and lights are cylinder-shaped. Tell students that the air blowers inside the PSA are also cylinder-shaped.

Have students visit the Student page of the Personal Satellite Assistant (PSA) Web site at <http://psa.arc.nasa.gov/> by clicking the "Students" tab to go to <http://psa.arc.nasa.gov/stud.shtml>. From this page, have students click the link to the "Using Diameter, Volume and Surface Area to Determine Dimensions of PSA's Computer." On this page, have students review the "Extension Problem video" at the bottom of the page under "Extension Problem." They can also review the text that summarizes this extension problem. This video clip discusses the problem of choosing a blower shape for the PSA.



PSA Components
Preliminary Engineering Sketch

Tell students they are going to investigate how the surface area changes when the dimensions of a cylinder change but the volume of the cylinder remains the same. Ask students what math relationships might help them understand this problem. Show cylinders that are tall and narrow and short and wide.

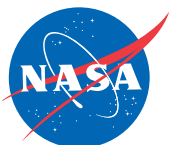
Show students a piece of clay in the shape of a cylinder.

Say: "We are going to keep the volume of the cylinder the same, and change the surface area."

Connect to students' previous knowledge of surface area.

Ask: "What is surface area?"

Listen carefully to students' responses. If they appear to be confused about surface area, talk about painting the surface and needing to know how much paint is necessary, or covering the surface with paper and needing to calculate the amount of paper you would need to cover the surface.



- ☐ Ask: "How can we calculate the surface area of this cylinder?"
- ☒ Answer: *The surface area is the sum of the areas of the circles at the top and bottom of the cylinder and the area around the side of the cylinder.*

Ask students for the formulas they will need for their calculations and write them on the board.

Volume of a Cylinder: $\pi \times \text{radius} \times \text{radius} \times \text{height}$

Surface Area of a Cylinder: $(2 \times \pi \times \text{radius} \times \text{radius}) + (2 \times \pi \times \text{radius} \times \text{height})$

Tell students that they will be investigating the following questions:

- How must the shape of the cylinder change in order for it to have the most surface area?
- Does a tall, narrow cylinder or a short, wide cylinder have more surface area?

Flatten the clay cylinder while still maintaining its cylindrical shape.

Ask: "What do you think has happened to the surface area? Has it increased or decreased?"

Accept all responses and ask students how they can check whether they are correct.

Review with students how to calculate ratios and how they are interpreted.

Explore

To each group, distribute one clay cylinder, rulers, calculators, and Student Handout sheets.

Ask students to measure the radius and height of the cylinder and then calculate its volume and surface area. They should also calculate the height to radius ratio. Students should record their results on the Student Handout sheet.

Challenge students to change the cylinder dimensions to find a cylinder with maximum and minimum surface areas.

If students have trouble finding cylinders with very large surface areas, suggest making cylinders that are long and narrow (like a straw) or short and flat (like a pancake). Students should be careful to make the clay cylinder as evenly distributed as possible. If the volumes of the various cylinders are very different from each other, the students are probably making uneven cylinders, or they are not making very accurate measurements. Instruct students to record their results on the Student Handout sheet. Tell students that they will share their results with the class.



Explain

Compare the maximum surface areas achieved by the students. Ask if the cylinders with the greatest surface areas were long and thin or short and flat. It is possible for the cylinder with the greatest surface area to be either. Ask students to look for a relationship between surface area and the ratios of height to radius. Guide them to conclude that cylinders tend to have the greatest surface areas when the ratios of their heights to radii are either very large or very small.

Compare the minimum surface areas achieved by the students. Ask what the cylinders with the smallest surface areas looked like. Were they tall and thin, short and flat, or somewhere in between?

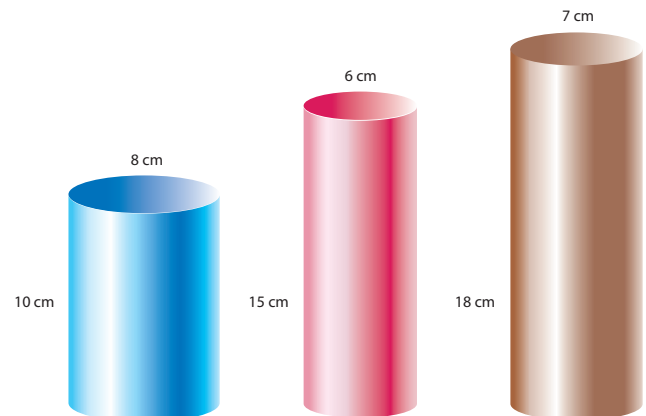
Extend

Ask students to make a recommendation to NASA on the cylinder shape of the blowers used to propel the PSA, which has a 20-cm diameter. Tell students that these blowers need to have the maximum surface area to release heat and will be mostly contained inside the PSA sphere, with only the tops exposed to the outside.

Bonus Problem

Tell students to consider ways in their lives that it is useful to understand how the height and radius affect the surface area and volume of a cylinder. Ask students to think about the following situation:

A restaurant offers three sizes of soda: small, medium, and large. Each cup is cylinder-shaped. The small soda costs \$1.00 and has a height of 10 centimeters and a diameter of 8 centimeters. The medium soda costs \$1.50 and has a height of 15 centimeters and a diameter of 6 centimeters. The large soda costs \$1.75 and has a height of 18 centimeters and a diameter of 7 centimeters.



Ask students to draw these three cups on a piece of paper. Without performing any calculations, have students draw each of the cylinders. Instruct students to hypothesize which of the three cups is the best value. Ask students to explain why they made their hypotheses.

Tell students to verify their hypotheses by calculating the volumes of the cylinders to determine which is the best value. Ask students to explain why the results came out as they did. Ask how tall a medium soda has to be in order for it to hold the same volume as a large soda.



Evaluate

As a class, create an assessment rubric for this activity. Suggested criteria for the rubric include:

- Accurate and consistent measurements of the volumes of cylinders.
- Appropriate calculations of surface areas and height-to-radius ratios.
- Appropriate reasoning methods to determine the relationship between the height-to-radius ratio and surface area of cylinders with equal volume.
- Correct assessment of the relationship between the height-to-radius ratio and surface area of cylinders with equal volume.
- Correct assessment as to which cylinder shape will fit in a sphere and produce maximum surface area.
- Clear oral reasoning as to why a cylinder-shape will or will not fit in a sphere while producing the maximum amount of surface area.
- Clear written presentation of results.
- Clear oral presentation of results.

Use the rubric to assess students' recommendations to NASA and solution to the soda of best value and ensure they have mastered the major concepts and math skills.



Student Handout

Formulas

Volume of a Cylinder: $\pi \times \text{radius} \times \text{radius} \times \text{height}$

Surface Area of a Cylinder: $(2 \times \pi \times \text{radius} \times \text{radius}) + (2 \times \pi \times \text{radius} \times \text{height})$

Surface Area vs. Volume of a Cylinder

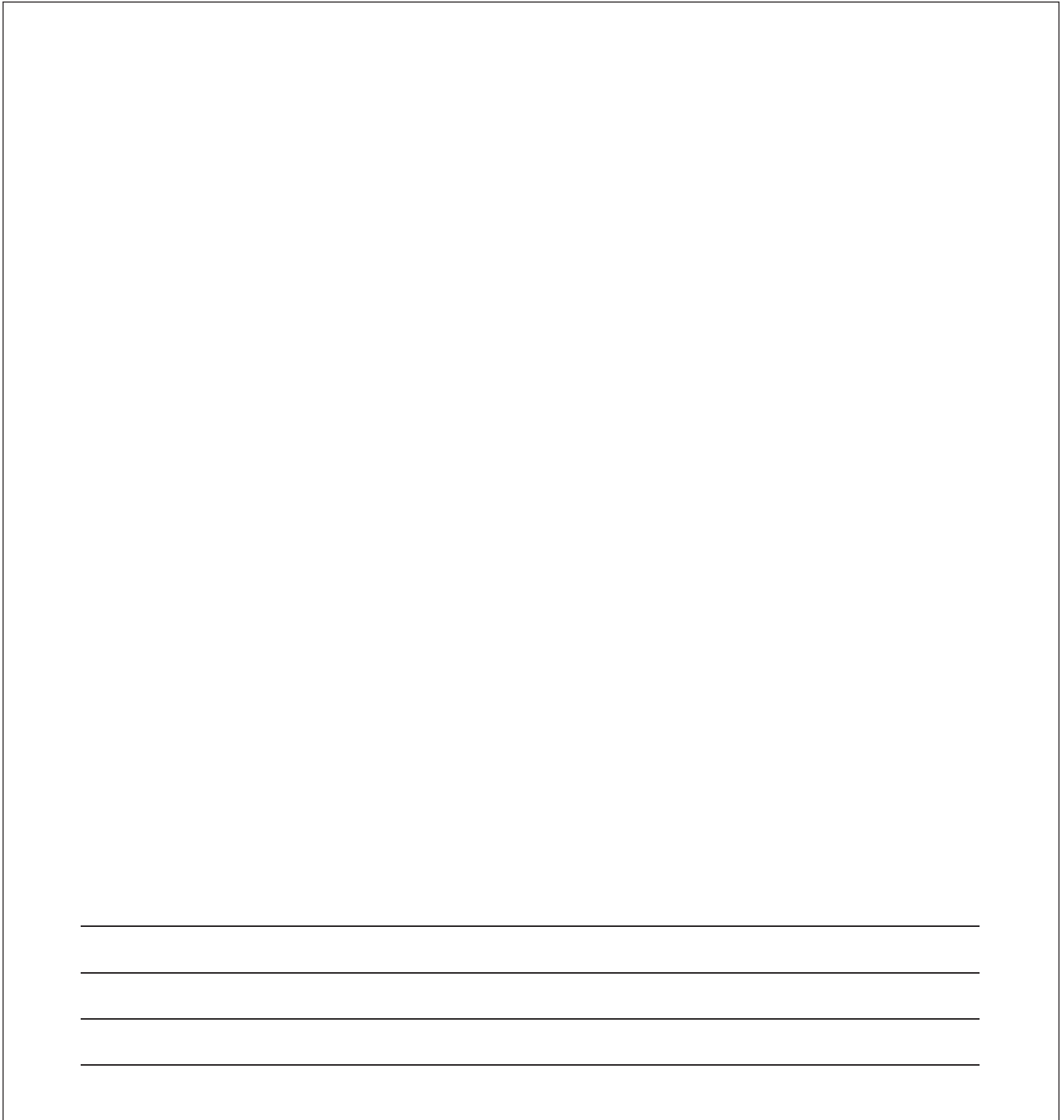
1. Using clay, create cylinders that will allow you to fill in the information below.

Height (centimeters)	Radius (centimeters)	Volume (cubic centimeters)	Surface Area (square centimeters)	Ratio of Height to Radius
8	1			

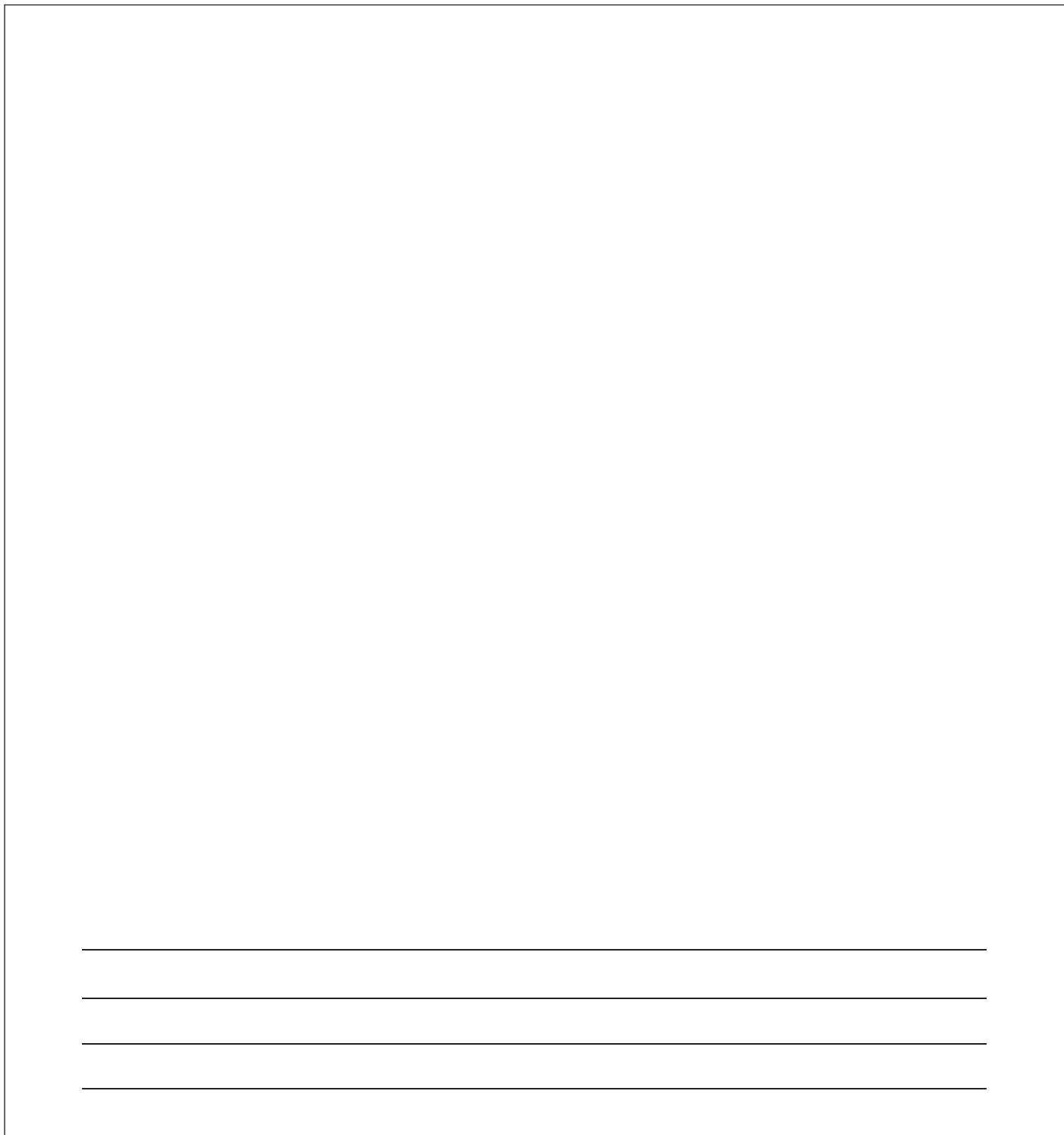
2. Describe the relationship between surface area and the radius of the cylinder.



3. Of the cylinders you created, what radius and height has the largest surface area? Draw and describe its shape.

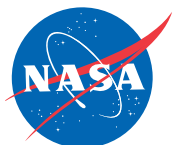


4. Of the cylinders you created, what radius and height has the smallest surface area? Draw and describe its shape.



Choosing Blower Shape for the PSA

1. NASA needs to use cylinder-shaped blowers to propel the PSA. The cylinders need to have the maximum surface area to release the most heat. The cylinders will mostly be contained inside the PSA sphere, with only the tops exposed to the outside. What recommendation would you make to NASA on the general cylinder shape of the blowers they should use to propel the PSA? Draw and describe its shape.



Bonus Problem

A restaurant offers three sizes of soda: small, medium, and large. Each cup is cylinder-shaped. The small soda costs \$1.00 and has a height of 10 centimeters and a diameter of 8 centimeters. The medium soda costs \$1.50 and has a height of 15 centimeters and a diameter of 6 centimeters. The large soda costs \$1.75 and has a height of 18 centimeters and a diameter of 7 centimeters.

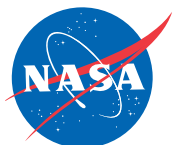
1. Draw each of the three cups on a separate piece of paper.
2. Without performing any calculations, explain which size you think is the best value.

3. Calculate the volumes of each of the cups in the table below. (Don't forget to convert the diameter to a radius by dividing the diameter in half!)

Height (centimeters)	Radius (centimeters)	Volume (cubic centimeters)	Ratio of Volume to Price
10			
15			
18			

4. Briefly explain why the results came out as they did.

5. How tall would a medium soda have to be in order for it to hold the same volume as a large soda?



Answer Key

Formulas

Volume of a Cylinder: $\pi \times \text{radius} \times \text{radius} \times \text{height}$

Surface Area of a Cylinder: $(2 \times \pi \times \text{radius} \times \text{radius}) + (2 \times \pi \times \text{radius} \times \text{height})$

Surface Area vs. Volume of a Cylinder

Students should find the surface area increases as the height-to-radius ratio is very large or very small. Surface area will be a minimum as the ratio nears 1, but it is not a minimum exactly at 1.

Choosing Blower Shape for the PSA

NASA needs to use cylinder-shaped blowers to propel the PSA. The cylinders need to have the maximum surface area to release the most heat. The cylinders will mostly be contained inside the PSA sphere, with only the tops exposed to the outside. What recommendation would you make to NASA on the general cylinder shape of the blowers they should use to propel the PSA? Draw and describe its shape.

Students should recommend cylinder shapes that have a very large or very small height-to-radius ratio. These would be either short, wide cylinders or tall, skinny cylinders. Both maximize the surface area. Really, the best recommendation would be the short, wide cylinders, because these would maximize the amount of space inside the PSA for other components.

Bonus Problem

A restaurant offers three sizes of soda: small, medium, and large. Each cup is cylinder-shaped. The small soda costs \$1.00 and has a height of 10 centimeters and a diameter of 8 centimeters. The medium soda costs \$1.50 and has a height of 15 centimeters and a diameter of 6 centimeters. The large soda costs \$1.75 and has a height of 18 centimeters and a diameter of 7 centimeters.

Calculate the volumes of each of the cups in the table below.

Height (centimeters)	Radius (centimeters)	Volume (cubic centimeters)	Ratio of Volume to Price
10	4	502.7	502.7
15	3	424.1	282.7
18	3.5	692.7	395.8

The best value is clearly the small soda, because it holds the most volume per dollar. In fact, a medium soda holds significantly less volume than a small, because the radius shrinks. The volume of a cylinder changes with the square of the radius, so a decrease in radius will change the volume much more substantially than a slight increase in height. For a medium soda to hold as much volume as a large, it would have to be 24.5 centimeters (over 9.5 inches) tall!



Sample Scoring Tool

4

- Calculations are correct and clearly presented.
- Students accurately interpret ratios.
- Reasoning is logical and clear explanations are provided.
- Oral and written presentations are clear.

3

- Most calculations are correct and attempts are made to present clearly.
- Students draw appropriate conclusions from ratios.
- Attempts are made to reason logically and provide clear explanations.
- Attempts are made to provide clear oral and written presentations.

2

- Some calculations are correct and attempts are moderately clear.
- Students have difficulty interpreting ratios.
- Explanations demonstrate limited logical bases.
- Oral and written presentation skills need improvement.

1

- Few calculations are correct and attempts are unclear.
- Students do not demonstrate adequate understanding of ratios.
- Explanations do not demonstrate understanding of lesson content.
- Oral and written presentations do not effectively express results or reasoning

